ALGORITHMS FOR CALCULATING GEODETIC HEIGHTS AND LATITUDES BY RECTANGULAR COORDINATES IN THE MERIDIAN ELLIPSE PLANE

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Owing to the widespread use of GNSS technologies in geodetic practice, the problem arises of transition from rectangular spatial coordinates of points to spatial geodetic coordinates, which are necessary for the transition to flat rectangular coordinates in the Gauss-Kruger projection. The authors proposed five algorithms for converting rectangular coordinates of points in the plane of the meridian ellipse into geodetic heights and latitudes. The first two algorithms are geometrically related to the intersection point of the ellipse with the normal passing through the point at which the rectangular spatial coordinates were obtained. The formulas of the other three algorithms are based on the geometric relationships of the point of intersection of the meridian ellipse with the straight line connecting the point with the center of curvature of the meridian. As a result of the experiments, deviations of the calculated latitudes and heights from the reference values of the given grid of geodetic coordinates were obtained. The formulas were tested not only for points under and on the earth's surface, but also outside the earth at different heights up to an altitude of 20,000 km.

Keywords: meridian ellipse, geodesic heights and latitudes, rectangular coordinates of the meridian ellipse, normal, center of curvature of the meridian, algebraic equations of the fourth degree, radius vector

REFERENCES

1. Afonin, K. F. (2011). *Vysshaya geodeziya. Sistemy koordinat i preobrazovaniya mezhdu nimi [Higher Surveying. Coordinate systems and transformations between them]*. Novosibirsk: SSGA Publ., 66 p. [in Russian].

2. Teleganov, N. A., & Elagin, A. V. (2004). *Vysshaya geodeziya i osnovy koordinatno-vremennykh system [Higher geodesy and the basics of coordinate-time systems]*. Novosibirsk: SSGA Publ., 238 p. [in Russian].

3. Laping, K. A. (1962). Calculation of coordinates and heights from the measured azimuths of normal sections and the angles of inclination of the chords at two starting points. *Izvestiya vuzov. Geodeziya i aerofotos"emka [Izvestiya vuzov. Geodesy and Aerophotosurveying]*, 1, 3–8 [in Russian].

4. Medvedev, P. A. (1993). Determining the latitude of a point by solving an algebraic equationвнения *Izvestiya vuzov. Geodeziya i aerofotos"emka [Izvestiya vuzov. Geodesy and Aerophotosurveying]*, 4, 66–72 [in Russian].

5. Medvedev, P. A. (2016). Research on methods for calculating the geodetic latitude and height of points on the earth's surface in rectangular coordinates. *Izvestiya vuzov. Geodeziya i aerofotos"emka [Izvestiya vuzov. Geodesy and Aerophotosurveying]*, 3, 24–28 [in Russian].

6. Medvedev, P. A., & Mazurov, B. T. (2016). Algorithms for the direct calculation of geo-desic latitude and geodesic height from rectangular coordinates. *Vestnik SGUGiT [Vestnik SSUGT]*, 3(34), 5–13 [in Russian].

7. Medvedev P. A., & Kenzheguzinova M. M. (2016). Calculation of the geodetic height from the rectangular spatial coordinates of the points on the earth's surface. *Vestnik Omskogo gosudarstvennogo agrarnogo universiteta [Bulletin of the Omsk State Agrarian University]*, 3(23), p. 146.

8. Medvedev, P. A. (2016). Mathematical models of transformations of spatial coordinates. *Geodeziya i kartografiya [Geodesy and Cartography]*, 3, 2–7 [in Russian].

9. Medvedev, P. A., Novorodskaya, M. V., & Sharov, S. A. (2017). Non-iterative algorithm for calculating geodetic latitude using spatial rectangular coordinates. *Vestnik Omskogo gosudarstvennogo agrarnogo universiteta [Bulletin of the Omsk State Agrarian University]*, 2(26), 60–64.

10. Maksimova, M. V. (2013). Century Transformations of coordinates in engineering and geodetic surveys. *Inzhenernye izyskaniya [Engineering Surveys]*, 2, 18–21 [in Russian].

11. Balandin, V. N., & Bryn, M. Ya., Imshenetskiy, S. P., Matveev, A. Yu., & Yuskevich, A. V. (2006). Algorithm for calculating the geodetic height from spatial rectangular coordinates. *Geodeziya i kartografiya [Geodesy and Cartography]*, 6, 15–16 [in Russian].

12. Kurchenko, L. A., Taran, V. V., & Shlapak, V. V. (2016). On the question of the trans-formation of geodesic rectangular coordinates into curved. *Izvestiya vuzov. Geodeziya i aerofotos"emka [Izvestiya vuzov. Geodesy and Aerophotosurveying]*, 3, 29–33 [in Russian].

13. Shanurov G. A., Manilova A. D. (2017). On the recalculation of spatial Cartesian coordi-nates into geodesics. *Izvestiya vuzov. Geodeziya i aerofotos"emka [Izvestiya vuzov. Geodesy and Aerophotosurveying]*, 1, 13–17 [in Russian].

14. Shanurov, G. A., Polovnev, O. V., & Manilova, A. D. (2015). Transformations of spatial coordinates during geodetic support of the scanning complex. *Izvestiya vuzov. Geodeziya i aerofotos"emka [Izvestiya vuzov. Geodesy and Aerophotosurveying]*, 1, 15–18 [in Russian].

15. Ogorodova, L. V. (2011). Joint calculation of the geodetic latitude and height of points on the surface of the earth. *Izvestiya vuzov. Geodeziya i aerofotos"emka [Izvestiya vuzov. Geodesy and Aerophotosurveying]*, 9, 11–15 [in Russian].

16. Gafiatulin, Kh. G., & Novoselov, O. G. (2017). Solutions of the geodesic problem of the inverse transformation of plane rectangular and polar coordinates, determined by a system of num-bers from one net-work to another through the projection of a conditionally auxiliary coordinate sys-tem. *Internet-zhurnal Naukovedenie [Internet Journal of Science]*, 3, 1–8 [in Russian].

17. Poleshenkov, V. N. (2011). Converting geocentric cartesian coordinates to geodetic coordinates. *Geodeziya i kartografiya [Geodesy and Cartography]*, 2, 15–19 [in Russian].

18. Bowring, B. R. (1985). The accuracy of geodetic latitude and height equations. *Survey Review*, *38*, 200–206.

19. Bowring, B. R. (1976). Transformation from spatial to geodetic coordinates. *Survey Review*, *23*, 323–327.

20. Vermeille H. (2002). Direct transformation from geocentric coordinates to geodetic coordinates. *Journal of Geodesy*, *76,* 451–454.

21. Afonin, K. F., & Trifonova, Yu. S. (2019). Determination of geodetic latitude by spatial rectangular coordinates using differential correction. In *Sbornik materialov Interekspo GEO-Sibir'-2019: Mezhdunarodnoy nauchnoy konferentsii: T. 1, No. 2. Geodeziya, geoinformatika, kartografiya i marksheyderiya [Proceedings of Interexpo GEO-Siberia-2019: International Scientific Conference: Vol. 1, No. 2. Surveying, Geoinformatics, Cartography and Mine Surveying]* (pp. 3–8). Novosibirsk: SSUGT Publ. [in Russian].

22. Afonin, K. F. (2020). Using differential corrections to calculate geodetic latitudes from spatial rectangular coordinates. *Vestnik SGUGiT [Vestnik SSUGT]*, *25*(1), 7–13 [in Russian].

23. Bronshtein, I. N., & Semendyaev, K. A. (1986). *Spravochnik po matematike dlja inzhenerov i uchashhihsja vuzov [A guide to mathematics for engineers and college students]* (13th ed.). Moscow: Nauka Publ., 544 p. [in Russian].

Received 23.06.2021